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A NOTE ON THE CONSTANT TERM METHOD TO MOCK THETA FUNCTIONS

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Dedicated to Prof. A.K. Agarwal on his 70^{th} ***Birth Anniversary***

Abstract: The main purpose of this paper is to show that certain mock theta functions can be expressed as constant terms in the Laurent series expansion of rational functions of theta functions. And, two identities can be proved by using the constant term method.

Keyword and Phrases: Mock theta function, Constant term method, Theta function, Hecke type series.

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1. Introduction

In his famous last letter to Hardy [20], Ramanujan introduced seventeen mock theta functions–four of order three, two groups five in each group of order five and three of order seven without giving an explicit definition. The mock theta functions are interpreted by Andrews and Hickerson [5] to mean a function $f(q)$ defined by a q-series which converges for $|q| < 1$ and satisfies the following two conditions:

(0) For every root of unity ξ, there is a theta function $\theta_\xi(q)$ such that the difference $f(q) - \theta_\xi(q)$ is bounded as $q \to \xi$ radially.

(1) There is no single theta function which works for all ξ; i.e., for every theta function $\theta(q)$ there is some root of unity ξ for which $f(q) - \theta_\xi(q)$ is unbounded as $q \to \xi$ radially.